What's All This E_b/N_o Stuff, Anyway?

By Jim Pearce (With Apologies to Bob Pease)

(This article originally appeared in the Fall 2000 issue of Spread Spectrum Scene Online, and can also be found at http://sss-mag.com/ebn0.html).

Anyone who has spent more than ten minutes researching digital communications has run across the cryptic notation E_b/N_o. Usually this shows up when discussing bit error rates or modulation methods. You may have a vague feeling that it represents something important about a digital communication system, but can't really put a finger on what or why. So let's take a look at just what this E_b/N_o thing is and why it's important.

First of all, how do you pronounce E_b/N_o? Most engineers that I know say "E bee over en zero," though some of the more fastidious ones say "E sub bee over en sub zero". At any rate, even though "No" is usually written with an "Oh" instead of a zero, it is not pronounced as the word "no".

E_b/N_o is classically defined as the ratio of Energy per Bit (Eb) to the Spectral Noise Density (No). If this definition leaves you with a empty, glassy-eyed feeling, you're not alone. The definition does not give you any insight into how to measure E_b/N_o or what it's used for.

E_b/N_o is the measure of signal to noise ratio for a digital communication system. It is measured at the input to the receiver and is used as the basic measure of how strong the signal is. Different forms of modulation -- BPSK, QPSK, QAM, etc. -- have different curves of theoretical bit error rates versus E_b/N_o as shown in Figure 1. These curves show the communications engineer the best performance that can be achieved across a digital link with a given amount of RF power.

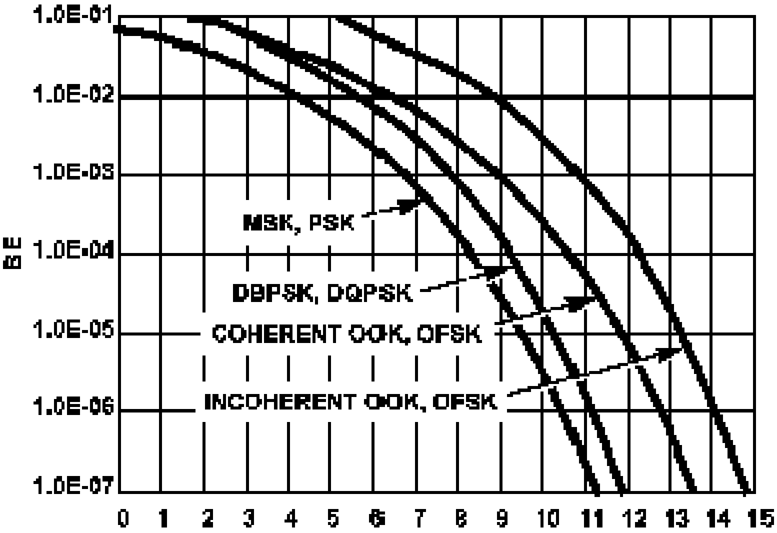

Figure 1. BER vs E_b/N_o
(Thanks, Intersil for this figure)

In this respect, it is the fundamental prediction tool for determining a digital link's performance. Another, more easily measured predictor of performance is the carrier-to-noise or C/N ratio.

So let's pretend that we are designing a digital link, and see how to use E_b/N_o and C/N to find out how much transmitter power we will need. Our example will use differential quadrature phase shift keying (DQPSK) and transmit 2 Mbps with a carrier frequency of 2450 MHz. It will have a 30 dB fade margin and operate within a reasonable bit error rate (BER) at an outdoor distance of 100 meters. Hold on to your hat here! Remember that when we play with dB or any log-type operation, multiplication is replaced by adding the dBs, and division is replaced by subtracting the dBs.

Our strategy for determining the transmit power is to:

- Determine E_b/N_o for our desired BER;
- Convert E_b/N_o to C/N at the receiver using the bit rate; and
- Add the path loss and fading margins.

We first decide what is the maximum BER that we can tolerate. For our example, we choose 10^{-6} figuring that we can retransmit the few packets that will have errors at this BER.

Looking at Figure 1, we find that for DQPSK modulation, a BER of 10^{-6} requires an E_b/N_o of 11.1 dB.

OK, great. Now we convert E_b/N_o to the carrier to noise ratio (C/N) using the equation:

$$\frac{C}{N} = \frac{Eb}{No} \bullet \frac{fb}{Bw}$$

Where:

- *fb* is the bit rate, and
- *Bw* is the receiver noise bandwidth. [***EDITOR'S NOTE: See Phil Karn's comment below concerning this equation.***]

So for our example, C/N = 11.1 dB + 10log(2x106 / 1x106) = 11.1 dB + 3dB = 14.1dB.

Since we now have the carrier-to-noise ratio, we can determine the necessary received carrier power after we calculate the receiver noise power.

Noise power is computed using Boltzmann's equation:

$$N = kTB$$

Where:

- k is Boltzmann's constant = 1.380650×10^{-23} J/K;
- T is the effective temperature in Kelvin, and
- B is the receiver bandwidth.

Therefore, N1 = (1.380650×10^{-23} J/K) * (290K) *(1MHz) = 4×10^{-15W} = 4×10^{-12}mW = -114dBm

Our receiver has some inherent noise in the amplification and processing of the signal. This is referred to as the receiver noise figure. For this example, our receiver has a 7 dB noise figure, so the receiver noise level will be: N = -107 dBm.

We can now find the carrier power as C = C/N * N, or in dB C = C/N + N:
C = 14.1 dB + -107dBm = -92.9 dBm

This is how much power the receiver must have at its input. To determine the transmitter power, we must account for the path loss and any fading margin that we are building in to the system.

The path loss in dB for an open air site is: PL = 22 dB + 20log(d/λ)

Where:

- PL is the path loss in dB;
- d is the distance between the transmitter and receiver; and
- λ is the wavelength of the RF carrier (= c/frequency)

This assumes antennas with no gain are being used. For our example,

PL = 22 dB + 20log(100/.122) = 22 + 20*2.91 =
22 + 58.27 = 80.27 dB

Finally, adding our 30 dB fading margin will give the required transmitter power:

P = -92.9 + 80.27 + 30 = 17.37 dBm = 55 mW

Our result, 55 mW, is well within a reasonable power level for spread spectrum links in the 2.4 GHz band. So we see that, in this example, our 100 meter range is a very reasonable expectation.

So, what is all this E_b/N_o stuff? Simply put, it's one of the "secrets" used by top RF design engineers to evaluate options for digital RF links, and is a crucial step in the design of systems that will meet performance expectations.

Comments from Phil Karn

From: Phil Karn To: Jim Pearce Sent: Monday, April 23, 2007 3:47 AM
Subject: Eb/No Explained

[Editor's Note: Phil is a Qualcomm engineer who is very well known in the radio community. His website is at www.ka9q.net, and has a number of articles of interest to electronics/wireless aficionados/practitioners.]

Hi Jim,

I found your article "What's All This Eb/No Stuff, Anyway?" while looking for references that would help me better explain this stuff.

It's a good paper, but I have a tiny little nit. Your first equation says:

C/N = Eb/No * fb/Bw, where

fb is the bit rate, and Bw is the receiver noise bandwidth

Usually I see this stated as

C/N = Eb/No * (R/B), where

R = bit rate B = channel bandwidth

I.e., "channel bandwidth" instead of "receiver noise bandwidth".

I see two problems with using receiver noise bandwidth in this equation. First, Eb/No is supposed to be a universal figure of merit for any kind of receiver, so it's measured at the receiver input terminals and is independent of anything inside that receiver. Different receiver designs for the same signal might use multiple filters with different shapes and bandwidths, but that would not affect the Eb/No of the signal at their inputs.

The other problem is that there's more than one definition of bandwidth. Noise bandwidth is just one of many. In fact, that's precisely why Eb/No is such a useful figure in the first place: it completely avoids arguments over the exact system bandwidth and/or which definition of bandwidth to use to measure it. No is the noise power spectral density in units of watts/Hz (or milliwatts/Hz), so the only filter bandwidth that's relevant is that of the spectrum analyzer being used to measure it.

The procedure I like for measuring Eb/No on the bench is to use an analyzer to measure the signal power with a resolution bandwidth wide enough to capture all of the signal. Then I turn off the signal source, turn on the noise generator, and measure the noise power on the analyzer with the resolution bandwidth set to the user data rate. (Naturally I have to ensure that both signal and noise swamp the analyzer's own noise).

Then I calculate Eb/No by simply subtracting the noise power measurement from the signal power measurement. Setting the analyzer RBW to the user data rate simplifies the calculation by causing the data rate and noise bandwidth terms to cancel and fall out of the equation.

I found your article while trying to explain to another person that his Eb/No measurement methods are wrong. This fellow claims to have invented a family of "ultra narrow band" modulation methods that are in fact ultra wide band (UWB) plus a very strong carrier that wastes most of the signal power. Among many other mistakes, he has fallen into the trap of confusing noise bandwidth with other, more relevant definitions of bandwidth, and his receivers have filters with noise bandwidths that are much smaller than the Nyquist rate. This is how he has fooled himself into thinking that his signals are narrow band.

Anyway, thanks again for the article you published way back in 2000.

Regards, Phil

Discussion Between Phil Karn and Steve Liang about E_b/N_o, July 2009

Printed here with permission from both Phil and Steve, and a real nuts and bolts discussion!

From: Steve Liang, Wednesday, July 22, 8:18 a.m.
Jim or Phil, Does either of you know where can I find BER vs. Eb/No waterfall plots, Excel spreadsheet even better, for different modulation schemes (BPSK, QPSK, 8PSK, and 16QAM)?

Phil - Does Qualcomm have BER vs. Eb/No data for different modulation schemes and for different Viterbi Turbo Coder rates?

Thanks,
Steve Liang
Sprint Nextel, San Francisco RFE

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From: Steve Liang, Wednesday, July 22, 2009 2:48 p.m.
Jim and Phil- I wasn't thinking clearly this morning. Don't worry about the 2nd question I have for Phil, it's not a valid question for Eb/No (1/2 rate or 1/3 rate FEC encode has not effect on BER.) Still like to have BER vs. Eb/No data though.

Thanks,
Steve Liang

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From: Phil Karn, Wednesday, July 22, 3:13 p.m.
BER vs Eb/No plots for the standard modulation techniques are in all the comm theory textbooks. The Wikipedia article for BPSK has plots, with derivations, of the bit error rate vs Eb/No for uncoded BSPK, QPSK, 8PSK and 16PSK.

FEC most definitely does affect the BER vs Eb/N0 plots. I can't speak to our own decoders off the top of my head, but plots for the common FEC schemes are also in the textbooks, certainly for the common Viterbi decoded codes.

Note that "The Viterbi Algorithm" is an algorithm for decoding convolutional codes. There's no such thing as a "viterbi code."

A Turbo code is a different kind of code made typically of two small convolutional encoders plus an interleaver. It is decoded by one of several methods, some including modified versions of the Viterbi algorithm that can produce "soft" decisions, i.e., estimates of the reliability of each output bit rather than just the algorithm's best guess as to its most likely value. This is important in decoding turbo codes since the process is iterative, feeding the results of one decoder into another until the data stops getting better.

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From: Steve Liang, Wednesday, July 22, 2009 7:50 p.m.
Phil, Thank you for the fast response. So, you do think FEC would reduce Eb/No requirement for a specific BER? I thought your comment to Jim's "Eb/No Explained" was "Eb/No is supposed to be a universal figure of merit for any kind of receiver, so

it's measured at the receiver input terminals and is independent of anything inside that receiver"?

My EVDO infrastructure provides mobile's Ec/Io measurements. I am trying to correlate the Ec/Io measurement to expected EVDO data throughput through:

$$\frac{C}{I} = \frac{E_b}{N_o}\frac{R_b}{W}$$

R_b = User data rate
W = CDMA carrier bandwith

And I can get $\frac{C}{I}$ from $\frac{E_c}{I_o}$

$$\frac{C}{I} = 10\log\frac{10^{\frac{E_c}{I_o}}}{1-10^{\frac{E_c}{I_o}}}$$

And user data rate will be:

$$R_b = W\frac{\frac{C}{I}}{\frac{E_b}{N_o}}$$

The only missing part for the above equation now is the E_b/N_0. I think the E_b/N_0 is a set point controlled by EVDO infrastructure vendor, it's not going to be easy to get it without run a deep debug trace at the system.

An alternative way to get user data throughput would be from the Qualcomm MSM chip, which I think is sending a 'DRC Index' up to CSM at network that maps to Payload size, Modulation scheme, FEC code rate, number of slots the payload will be transmitting, etc. Do you know where can I find the C/I to 'DRC Index' mapping table?

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From: Phil Karn, Monday, July 27, 2009 8:27 p.m.
You asked if I think FEC would reduce the Eb/No requirement for a specific BER. Yes, that's precisely the purpose of using FEC. It allows you to reduce the required Eb/No closer to the theoretical limit, which is -1.6 dB for infinite bandwidth. For

limited bandwidth, the required Eb/No is higher; e.g. at a ratio of 1 bit/sec per hertz of bandwidth, the absolute minimum Eb/No is 0 dB.

You said, "I thought your comment to Jim's *Eb/No Explained* was 'Eb/No is supposed to be a universal figure of merit for any kind of receiver, so it's measured at the receiver input terminals and is independent of anything inside that receiver?" That's right, the Eb/No of a particular signal is set at the input terminals of a receiver. The Eb/No at that point cannot be affected by anything inside that receiver. Note that this is distinct from the minimum *required* Eb/No needed to make that receiver actually work.

The Ec/I0 ratio is actually the Ec/(I0+N0) ratio; the interference and thermal noise are summed, but in practice the interference is much stronger so the noise can usually be disregarded. In any event, in a CDMA system interference looks like noise so they can be considered the same thing.

The CDMA reverse link uses a closed-loop power control scheme to try to set the Eb/N0 at the base receiver to a specific value that enables the receiver to just work well. There's no point in making the signal any stronger, as that would just increase the interference to other users. The cell receiver measures the Eb/N0 and tells the mobile transmitter to go up or down in 1 dB steps.

The CDMA forward link also uses closed loop power control, but it is much less critical to the operation of the system and to be honest I don't know as much about it. It's less critical because, unlike the CDMA reverse link, the CDMA forward link isn't a multiple access link. Interference exists from other cell site transmitters, but it's not as significant as the interference from other mobile transmitters on the reverse link because the cell site is trying to demodulate all those mobile transmitters at once. The mobile is not trying to demodulate multiple cell site transmitters at once except when it's in soft handoff, and in that case it's okay for one cell to be much stronger than the other because they're both sending the same information.

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From: Steve Liang, sent Tuesday, July 28, 2009 2:34 p.m.
Phil -- Thank you for the detailed explanation. The theoretical Eb/No for infinite bandwidth and the Eb/No for 1 bit/sec per hertz of bandwidth are new to me. Would you mind letting me know how you get those numbers? I failed to get those numbers trying to use the following relations:
Shannon's Channel Capacity Theorem:

$$R_b = BW \log_2\left(1 - \frac{C}{I}\right)$$

And

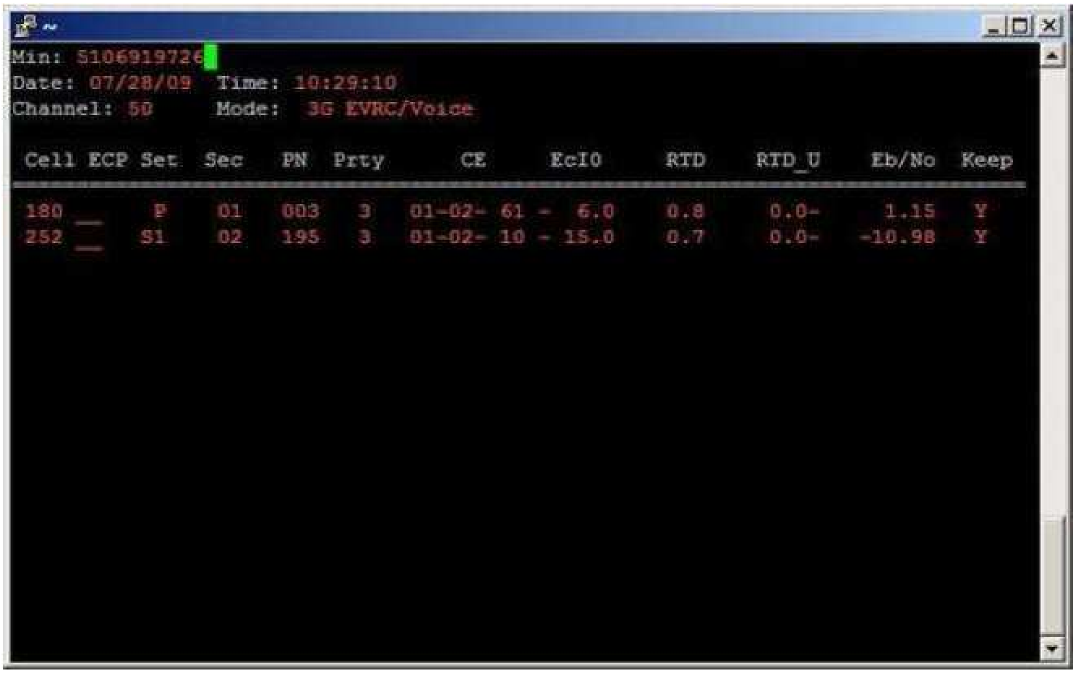
$$\frac{C}{I} = \frac{E_b}{N_o} \frac{R_b}{BW}$$

Gives

$$\frac{E_b}{N_o} = \frac{\left(2^{\frac{R_b}{BW}} - 1\right)}{\frac{R_b}{BW}}$$

I am not getting your numbers when I take limit for BW → infinity?

While we are on this topic, can you help me with this mystery I always have with Lucent call trace output. It gave unrealistic Eb/No values (see example below):



Last question, sorry for so many them – Does Qualcomm have a published table/chart for Eb/No vs. modulation scheme + different code rate for a specific BER, FER, or PER?

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From: Phil Karn, Tuesday, July 28, 2009 1:13 PM
You wrote: "The theoretical Eb/No for infinite bandwidth and the Eb/No for 1 bit/sec per hertz of bandwidth are new to me. Would you mind letting me know how do you

get those numbers?"

That's just the Shannon limit!
$C = B * \log2(1+S/N)$

C is the channel capacity in bits/sec, B is the channel bandwidth (you can't send *anything* outside it), S is the received signal power, and N is the received noise level, again both being confined to bandwidth B.

You can rework Shannon's basic equation as follows:
$C = B * \log2(1+(Eb/N0) * (C/B))$

This substitution works because S, the signal power, is just the energy per bit times the bit rate C, and N, the noise power, is the bandwidth times the noise power spectral density N0. Now we see the explicit relationship between channel capacity and R/B, the modulation efficiency in bits/sec / hz. You can rearrange this to $Eb/No >= (S/N) / (C/B) = (2^{(C/B)} - 1) / (C/B)$

In other words, this is the absolute minimum Eb/No ratio required for a link with a given C/B (bits/sec/Hz) ratio. This is only a theoretical minimum; any real system will do worse, though with modern FEC (turbo coding) you can now often get within a dB of the Shannon limit.

If you plug in C/B = 1 bit/sec/Hz, then you get a Eb/No = 1, which is 0 dB. If we decrease C/B, i.e., use excess bandwidth, the required Eb/No decreases but not forever. If you take the mathematical limit as C/B approaches infinity you get an answer of ln(2) or 0.693 or -1.6 dB. This is the famous "Shannon bound" that says no communication system, even if it has infinite bandwidth, can operate below an Eb/No of -1.6 dB.

That's where those figures came from.

Look closely at that exponential. As you increase C/B, the bits/sec/Hz, the minimum required Eb/No to make the link work *even in theory* increases *exponentially*. Yes, it's diminished somewhat by the (C/B) term in the denominator, but that doesn't grow nearly as fast as the exponential in the numerator. The C/B factor accounts for the fact that when you transmit a group of bits as a single symbol, that symbol can use the combined energies of all those bits.

Anyway, this is why you don't see systems running hundreds or even tens of bits/sec per hertz of bandwidth. The densest one I know of in widespread use is 256QAM in digital cable modems and TV. That's 256 possible values per symbol, or 8 bits/symbol, or 8 bits/sec / Hz. This is possible only on a hybrid fiber/coax cable network with few amplifiers and low distortion.

The equation you have with Ro in the result is for something called the "R0", or the computational cutoff rate, a value below the Shannon capacity C. The discovery of Turbo codes made this formula obsolete.

R0 was once thought to be the practical limit for any real FEC running on hardware we could actually build, and it would be impractical to exceed it. This was true for sequential decoding, one of the earliest of the powerful FEC decoding techniques, but sequential decoding was displaced by Viterbi decoding, which has since given way to Turbo decoding although it still uses a modified form of the Viterbi algorithm.

You also asked if Qualcomm has a published table/chart for Eb/No vs. modulation scheme+ different code rate for a specific BER, FER, or PER. I'm not sure, but this might be in the various EIA/TIA standards for our stuff. In particular, the 1xEV-DO spec might have specs for the required Eb/No for each of the many modulation modes and data rates.

I can't really comment on the channel analyzer picture since I don't know anything about your analyzer. If I had to guess I'd say those two Eb/No figures were for the two channels participating in a CDMA soft handoff. In this case, the receiver combines their energies before decoding. In this way it can often get enough energy to decode a bit that would not be possible with just the data from one cell site receiver. To add the two Eb/No ratios you have to first convert them back to linear:

1.15 dB -> 1.303
-10.98 dB -> 0.08
1.303 + 0.08 = 1.383 -> 1.41 dB. Not much better, but still better than just the one channel. What did the voice sound like? It might actually work at this low level, though it wouldn't sound great.

Hope this helps.

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From: Steve Liang, sent Tuesday, July 28, 2009 4:24 p.m.
Phil, thank you so much, this really answered the mystery I had about Eb/No. While I was taking my after lunch I did notice I made a mistake in Shannon's: Should be (1+S/N): so if we can have infinite signal power or bandwidth we can achieve infinite data rate.

Also the S/N and Eb/No are log values; I should convert them to decimal for the calculation. Good to talk to you, I have learned a lot.

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